BSc. Semester End Examination, January/February 2022

Semester: III
Subject: Mathematics
Title: Combinatorics (Elective)
Duration: 2 hours                                                    Max Marks: 60

**Instructions:** 1. All questions are compulsory. However internal choice is applicable.
2. Figures to the right indicate full marks.
3. Justify all responses.

**Q1. Answer <u>ANY FOUR</u> the following questions:**                              **(12)**

A. In how many ways can a man, woman, boy and girl can be selected from 5 men, 6 women, 2 boys and 4 girls?

B. There are 740 pigeons in a collection of pigeon holes. If each pigeon hole contains 5 pigeons, then what are the number of pigeon holes?

C. What are the number of 4-letter words that can be formed by using each of the letters a,b,c,d,e atmost once?

D. How many possibilities are there for 8 nonattacking rooks on an 8-by-8 chess board?

E. Expand $(x + y)^5$ using the binomial theorem.

**Q2. Answer <u>ANY TWO</u> the following questions:**                              **(12)**

A. Find the numbers between 1 and 1000, inclusive, that are not divisible by 5, 6 and 8.

B. Determine the number of 10-combinations of the multiset $\{3.a, 4.b, 5.c\}$.

C. Determine the number of solutions of the equation $x_1 + x_2 + x_3 + x_4 = 14$ in nonnegative integers $x_1, x_2, x_3$ and $x_4$ not exceeding 8.

**Q3. Answer <u>ANY THREE</u> the following questions:**                              **(12)**

A. From the integers 1,2,…., 200, we choose 101 integers. Show that among the integers chosen, there are two such that one of them is divisible by the other.

B. Show that if $n + 1$ integers are chosen from the set $\{1,2,….,2n\}$ then there are always two which differ by 1.

C. A child watches TV atleast one hour each day for 7 weeks but never more than 11 hours in any one week. Prove that there is some period of consecutive days in which the child watches exactly 20 hours of TV.

D. Prove that of any five points chosen within a square of side length 2, there are two whose distance apart is at most $\sqrt{2}$.

**Q4. Answer <u>ANY TWO</u> of the following questions:**                              **(8)**

A. How many odd numbers between 1000 and 9999 have distinct digits.

B. Ten people, including two who do not wish to sit next to one another are to be seated at a round table. How many circular seating arrangements are there?

C. What are the number of permutations of the letters in the word MISSISSIPPI.

**Q5. Answer <u>ANY TWO</u> of the following questions:** (4)

A. In how many ways can we put 10 green balls into 6 boxes.

B. Write the permutation 321564 of 123456 as a combination of its disjoint cycles.

C. What are the number of ways the letters M,A,D,I,S,O,N can be written so that the word spelled disagrees completely with the spelling of the word MADISON?

**Q6. Answer <u>ANY TWO</u> of the following questions:** (12)

A. Solve $h_n = 2h_{n-1} + 3^n$, $(n \geq 1)$, $h_0 = 2$.

B. Solve $h_n = 3h_{n-1} - 4^n$, $(n \geq 1)$, $h_0 = 2$.

C. Consider the multiset S={3.a, 2.b, 4.c} of nine objects of three types. Find the number of 8-permutations of S.

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